CALCULATION OF HEAT TRANSFER IN THREE-COMPONENT SYSTEMS

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An account is given of a method of calculating heat transfer in equipment with three heat transfer agents under conditions of variable heat transfer coefficients and surfaces.

In industry there are various forms of equipment in which three heat transfer agents play a part in heat transfer. Such equipment is used, in particular, in cases where mixing of the agents is not allowable, due to the possibility of undesirable chemical reactions or to the danger of explosion [1], when heat transfer takes place between two agents, with heat loss to the surrounding medium [2], when an intermediate agent is used in various technical processes [3], and during the firing of materials in the disperse state [4].

An analytical solution has been given in [1,2,5,9]. In these papers the assumption is made that the heat transfer coefficients and surfaces are constant, which is justified in a number of cases. However, if a disperse material plays a part in the heat transfer, for example, in pneumatic transport equipment, the heat transfer surface is a variable quantity. An example is the equipment illustrated in the figure.

The gas and the finely divided material are moving vertically upward, while the larger particles move in the opposite direction (a). Motion of all three agents in one direction is possible (b). It is well known that in in this case both the heat transfer surface and the coefficient are functions of the coordinate x.

For the heat transfer process represented in the figure, the system of equations for zero-gradient heating of the particles has the form

$$dt' = -\frac{\alpha_2}{W_1}(t'-t'')\,dF_2 - \frac{\alpha_3}{W_1}(t'-t''')\,dF_3, \quad (1)$$

$$dt'' = \frac{\alpha_2}{W_2}(t'-t'')\,dF_2, \quad (2)$$

$$dt''' = \mp\frac{\alpha_3}{W_3}(t'-t''')\,dF_3. \quad (3)$$

The upper sign in (3) and in all the following expressions corresponds to the scheme of (a), and the lower sign to that of (b).

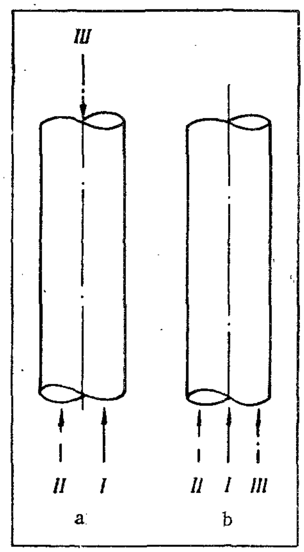
In these equations the heat transfer coefficients α_2 and α_3 are functions of the surfaces F_2 and F_3, respectively.

Since the heat transfer coefficient depends on the relative velocity of the gas and the material, the maximum values of the coefficients α_{2i} and α_{3i} will be at the initial section of the equipment ($F_2|_{x=0} = 0$).

A preliminary analysis showed that the variation of heat transfer coefficient with the coordinate x may be represented in the form of the following relations:

$$\alpha_2 = \alpha_{2i}\exp(-c_2F_2), \quad \alpha_3 = \alpha_{3i}\exp(-c_3F_2). \quad (4)$$

It should be noted that in a certain range relations (4) approximate even to a linear variation of heat transfer coefficients, with sufficient accuracy.



Motion of the heat transfer agents; I) gas; II) finely divided materials; III) coarse particles.

As has been mentioned, the heat transfer surfaces also change with change of x, and then the relation between them may be represented in the linear form

$$F_3 = bF_2. \quad (5)$$

We shall introduce the following notation:

$$\Theta' = (t'-t_i')/(t_i'-t_i''); \quad (6)$$

$$\Theta'' = (t''-t_i')/(t_i'-t_i''); \quad (7)$$

$$\Theta''' = (t'''-t_i')/(t_i'-t_i''); \quad (8)$$

$$\frac{\alpha_{2i}}{W_1}F_2 = X; \quad (9)$$

$$\frac{\alpha_{2i}}{\alpha_{3i}}\frac{dF_3}{dF_2} = \beta. \quad (10)$$

Allowing for (6)–(10), system (1)–(3) takes the form

$$\frac{d\Theta'}{dX} = -(\Theta'-\Theta'')\exp(-c_4X) - \beta(\Theta'-\Theta''')\exp(-c_5X), \quad (11)$$

$$\frac{d\Theta''}{dX} = R_{12}(\Theta'-\Theta'')\exp(-c_4X); \quad (12)$$

$$\frac{d\Theta'''}{dX} = \mp\beta R_{13}(\Theta'-\Theta''')\exp(-c_5X). \quad (13)$$

Here

$$c_4 = c_2 \frac{W_1}{\alpha_{2i}}, \quad c_5 = c_3 \frac{W_1}{\alpha_{2i}}.$$

Solutions of system (11)–(13), with respect to Θ'' leads, after some transformations, to the following equations:

$$\frac{d^3\Theta''}{dX^3} \mp [A \exp(-c_5X) + B\exp(-c_4X) + C] \times \frac{d^2\Theta''}{dX^2} \mp \{D\exp[-(c_4+c_5)X] + c_5 B\exp(-c_4X) + c_4 A\exp(-c_5X) + L\}\frac{d\Theta''}{dX} = 0, \quad (14)$$

where

$$A = \beta(R_{13} \mp 1); \quad B = \mp(1 + R_{12}); \quad C = \mp(c_5 + c_4);$$
$$D = \beta R_{12}R_{13}(1 \mp R_{31} + R_{21}); \quad L = \mp c_4(c_5 + c_4).$$

By means of the substitution

$$Y = \frac{d\Theta''}{dX} \quad (15)$$

Eq. (14) is reduced to the form

$$\frac{d^2Y}{dX^2} \mp P_1(X)\frac{dY}{dX} \mp Q_1(X)Y = 0, \quad (16)$$

where $P_1(X)$ and $Q_1(X)$ are the coefficients of $d^2\Theta''/dX^2$ and $d\Theta''/dX$, respectively.

Let us formulate the boundary conditions.

The first establishes the value of the temperature of agent II in the initial section

$$t''|_{X=0} = t''_1 \text{ or from (7) } \Theta''|_{X=0} = -1. \quad (17)$$

We obtain the second boundary condition from (12),

$$\left.\frac{d\Theta''}{dX}\right|_{X=0} = R_{12} \quad \text{or} \quad Y|_{X=0} = R_{12}. \quad (18)$$

Boundary conditions (17) and (18) are valid for both cases. From (13) we obtain

$$\Theta' = \frac{d\Theta''}{dX} R_{21}\exp(c_4X) + \Theta''. \quad (19)$$

Substituting (19) into (11), we obtain

$$\frac{d^2\Theta''}{dX^2} = -[c_4 + R_{12}\exp(-c_4X)]\frac{d\Theta''}{dX} - R_{12}\exp(-2c_4X) \times (\Theta' - \Theta'') - \beta R_{12}\exp[-(c_4+c_5)X](\Theta' - \Theta'''). \quad (20)$$

Hence we obtain for the counterflow case (a)

$$\left.\frac{dY}{dX}\right|_{X=0} = \left.\frac{d^2\Theta''}{dX^2}\right|_{X=0} = R_{12}(\beta\Theta'''_f - c_4 - R_{12} - 1), \quad (21)$$

and for the direct-flow case (b)

$$\left.\frac{dY}{dX}\right|_{X=0} = \left.\frac{d^2\Theta''}{dX^2}\right|_{X=0} = R_{12}(\beta\Theta'''_i - c_4 - R_{12} - 1). \quad (21a)$$

We shall seek a solution of (16) in the form of a power series

$$Y = \sum_{n=0}^{\infty} s_n X^n. \quad (22)$$

According to Cauchy's theorem [6], since the coefficients of the first and second derivatives in (14) do not have singular points, the series (22) converges for any X.

We represent the coefficients of the derivatives in (14) in the form of the following series:

$$P_1(X) = C + A\exp(-c_5X) + B\exp(-c_4X) = C + \sum_{m=0}^{\infty}(Ac_5^m + Bc_4^m)\frac{X^m}{m!}(-1)^m = a_0 + \sum_{m=1}^{\infty} a_m X^m, \quad (23)$$

where

$$a_0 = A + B + C; \quad a_m = (Ac_5^m + Bc_4^m)\frac{1}{m!}(-1)^m; \quad (23a)$$

$$Q_1(X) = L + D\exp[-(c_4+c_5)X] + c_5 B\exp(-c_4X) + c_4 A\exp(-c_5X) = L + \sum_{m=0}^{\infty}[D(c_4+c_5)^m + c_5Bc_4^m + c_4Ac_5^m]\frac{X^m}{m!}(-1)^m = b_0 + \sum_{m=1}^{\infty} b_m X^m, \quad (24)$$

where

$$b_0 = L + D + c_5B + c_4A;$$
$$b_m = [D(c_4+c_5)^m + c_5Bc_4^m + c_4Ac_5^m]\frac{(-1)^m}{m!}. \quad (24a)$$

The series (23) and (24) are absolutely convergent.

As is well known, the general solution of (16) has the form

$$Y = C_1Y_1 + C_2Y_2. \quad (25)$$

By substituting (22)–(24) into (16) and determining the coefficients s_n of (22) by the method of undetermined coefficients, under the following constraints:

$$Y_1|_{X=0} = 1, \quad Y'_1|_{X=0} = 0,$$
$$Y_2|_{X=0} = 0, \quad Y'_2|_{X=0} = 1, \quad (26)$$

we find particular solutions Y_1 and Y_2 of (25). The constants of integration C_1 and C_2 are determined from (25) with boundary conditions (18) and (21), and (21a) with account for (22), respectively,

$$C_1 = R_{12}. \quad (27)$$

For the counterflow case

$$C_2 = R_{12}(\beta\Theta'''_f - c_4 - R_{12} - 1), \quad (27a)$$

and for parallel-flow

$$C_2 = R_{12}(\beta\Theta'''_i - c_4 - R_{12} - 1). \quad (27b)$$

We thus obtain

$$Y = C_1 \sum_{n=0}^{\infty} s_n^{(1)} X^n + C_2 \sum_{n=0}^{\infty} s_n^{(2)} X^n. \quad (28)$$

According to (15), by integrating (28), we find an expression for Θ'',

$$\Theta'' = C_1 \sum_{n=0}^{\infty} s_n^{(1)} \frac{X^{n+1}}{n+1} + C_2 \sum_{n=0}^{\infty} s_n^{(2)} \frac{X^{n+1}}{n+1} + C_3, \quad (29)$$

where C_3 is determined from boundary conditions (17),

$$C_3 = -1. \quad (30)$$

The coefficients in (29) were determined by the method described above, and are equal, respectively, to

$$s_0^{(1)} = 1, \quad s_0^{(2)} = 0,$$
$$s_1^{(1)} = 0, \quad s_1^{(2)} = 1,$$
$$s_{n+1}^{(1),(2)} = \pm \frac{1}{n(n+1)} \left[\sum_{i=1}^{n} i s_i a_{n-i} + \sum_{j=i-1}^{n-1} s_j b_{n-(j+1)} \right], \; n = 1, 2, 3... \quad (31)$$

For $X \leq 0.5$ the series in (29) converge quite rapidly, and it is sufficient to take only the first few terms.

If $X > 0.5$, we may replace X in (29) by $X - X_0$, where $X - X_0 \leq 0.5$. In this case all the foregoing arguments remain in force, only the reference point being changed.

It should be noted that to obtain enhanced heat transfer in the weightless state, there has been a tendency recently to build equipment in which so-called acceleration sections [8] are included. As a rule, the extent of the effective part of these is small, and, in practice, in the majority of cases $X \leq 0.5$. The greatest change in heat transfer coefficient α occurs in the acceleration sections, and neglect of this change in α may lead to substantial errors.

From (12), allowing for (15), we obtain an expression for determining the dimensionless temperatures of the gases

$$\Theta' = \Theta'' + Y R_{21} \exp(c_4 X). \quad (32)$$

The dimensionless temperature of agent III is determined from (11),

$$\Theta''' = \Theta' + \frac{1}{\beta} \exp(c_5 X) \frac{d\Theta'}{dX} + \frac{1}{\beta} \exp[(c_5 - c_4) X](\Theta' - \Theta''). \quad (33)$$

By substituting the value of $d\Theta'/dX$ from (32) into (33), and taking account of (15) and (28), we obtain, after some transformations,

$$\Theta''' = \Theta' + \Big\{ Y[1 + R_{21} c_4 \exp(c_4 X)] + R_{21} \exp(c_4 X) \Big(c_1 \sum_{n=1}^{\infty} n s_n^{(1)} X^{n-1} + c_2 \sum_{n=1}^{\infty} n s_n^{(2)} X^{n-1} \Big) + \exp(-c_4 X)(\Theta' - \Theta'') \Big\} \frac{1}{\beta} \exp(c_5 X). \quad (34)$$

For the counterflow case the value of Θ_f''' appearing in (29), (32), and (34) may be determined, because of (27a), as follows.

The heat balance equations for the variant of heat agent motion in question (counterflow) has the form

$$t_i' - t_f' = -R_{21}(t_i'' - t_f'') - R_{31}(t_i''' - t_f'''). \quad (35)$$

Going over to dimensionless quantities according to relations (6)–(8), and making certain transformations, we obtain from (35)

$$\Theta_f''' = \Theta_i''' - R_{13}\Theta_f' - R_{21}R_{13}(1 + \Theta_f''), \quad (36)$$

where

$$\Theta_i''' = (t_i''' - t_i')/(t_i' - t_i'').$$

Substituting into (36) the values of Θ_f'' and Θ_f' from (29) and (32) with $X = X_f$, and expanding the resulting equality with respect to Θ_f''', we obtain

$$\Theta_f''' = \Big[\Theta_i''' - R_{13}(1 + R_{12}) \sum_{n=0}^{\infty} s_n^{(1)} \frac{X_f^{n+1}}{n+1} - R R_{13}(1 + R_{21}) \sum_{n=0}^{\infty} s_n^{(2)} \frac{X_f^{n+1}}{n+1} + R_{13} - R_{13} \exp(c_4 X_f) \sum_{n=0}^{\infty} s_n^{(1)} X_f^n - R R_{21} R_{13} \exp(c_4 X_f) \sum_{n=0}^{\infty} s_n^{(2)} X_f^n \Big] \times \Big[1 + R_{13}\beta(1 + R_{12}) \sum_{n=0}^{\infty} s_n^{(2)} \frac{X_f^{n+1}}{n+1} + R_{13}\beta \exp(c_4 X_f) \sum_{n=0}^{\infty} s_n^{(2)} X_f^n \Big]^{-1}, \quad (37)$$

where

$$R = -R_{12}(c_4 + R_{12} + 1).$$

By way of example, we shall examine a heat exchange equipment in which the motion of the heat transfer agent corresponds to scheme (a), it being the case that $t_i' > t_i''$, $t_i' > t_i'''$, $t_i'' = t_i'''$. Suppose that we are required to determine the final temperatures of agents I and II, given that $R_{12} = 2.65$; $R_{13} = 1$; $c_4 = c_5 = 0.7$; $\beta = 2$; $X = 0.1$; $\Theta_i''' = -1$; $t_i = 400°$ C; $t_i'' - t_i''' = 20°$ C.

From (23a) and (24a) we determine a_m and b_m, taking into account the notation of (14),

$$a_0 = -5.74; \quad a_1 = 2.6; \quad a_2 = -0.91; \quad a_3 = 0.21;$$
$$b_0 = -1.6; \quad b_1 = -1; \quad b_2 = 1.35; \quad b_3 = -0.78.$$

According to (31)

$$s_0^{(1)} = 1; \quad s_1^{(1)} = 0; \quad s_2^{(1)} = -0.8; \quad s_3^{(1)} = 1.37;$$
$$s_0^{(2)} = 0; \quad s_1^{(2)} = 1; \quad s_2^{(2)} = -2.9; \quad s_3^{(2)} = 5.7.$$

From (37) we find $\Theta_i''' = -0.83$.

From (30), (27), and (27a) we determine

$$C_3 = -1; \quad C_1 = 2.64; \quad C_2 = -15.8.$$

From (29)

$$\Theta''|_{X=0,1} = -0.8.$$

From (32), taking into account (28),

$$\Theta'|_{X=0.1} = -0.25.$$

From relations (6)–(8), the final temperatures of agents I, II, and III are equal, respectively, to

$$t_f' = 305°; \quad t_f'' = 95°; \quad t_f''' = 85°.$$

There is considerable practical interest in the case of heat transfer between two heat transfer agents with a surrounding medium at constant temperature, i.e., where losses must be taken into account. If the surrounding medium is the third agent, it is clear that $R_{13} \to 0$. Then, instead of (11)–(13), we have the following system of equations:

$$\frac{d\Theta'}{dX} = -(\Theta' - \Theta'')\exp(-c_4X) - \beta(\Theta' - \Theta_c'''), \quad (38)$$

$$\frac{d\Theta''}{dX} = \mp R_{12}(\Theta' - \Theta'')\exp(-c_4X), \quad (39)$$

$$\frac{d\Theta'''}{dX} = 0, \quad (40)$$

where

$$\beta = \frac{k}{\alpha_{2i}} b.$$

As before, the upper sign here refers to counterflow, and the lower to parallel flow.

From (38)–(40) we obtain

$$\frac{d^2\Theta''}{dX^2} + P_2(X)\frac{d\Theta''}{dX} + Q_2(X)\Theta'' = R(X), \quad (41)$$

where for parallel flow

$$P_2(X) = c_4 + \beta + (1 + R_{12})\exp(-c_4X),$$
$$Q_2(X) = \beta R_{12}\exp(-c_4X);$$

for counterflow

$$P_2(X) = c_4 + \beta + (1 - R_{12})\exp(-c_4X),$$
$$Q_2(X) = -\beta R_{12}\exp(-c_4X);$$

and for both cases

$$R(X) = \beta R_{12}\Theta_c'''\exp(-c_4X).$$

Let us write down the boundary conditions. From (7) the first boundary condition is for parallel flow

$$\Theta''|_{X=0} = -1, \quad (42)$$

for counterflow

$$\Theta''|_{X=0} = \Theta_f''. \quad (43)$$

We obtain a second boundary condition from (39), for parallel flow

$$\left.\frac{d\Theta''}{dX}\right|_{X=0} = R_{12}, \quad (44)$$

for counterflow

$$\left.\frac{d\Theta''}{dX}\right|_{X=0} = R_{12}\Theta_f''. \quad (45)$$

We seek a solution of (41) in the form of a particular integral which will satisfy conditions (42)–(45). In the same way as before, we may represent this solution in power series form,

$$\Theta'' = \sum_{n=0}^{\infty} s_n X^n. \quad (46)$$

The coefficients $P_2(X)$, $Q_2(X)$ and the free term $R(X)$, after the expansion in series, take the following form:

$$P_2(X) = A + B\exp(-c_4X) = A + B\sum_{m=0}^{\infty}\frac{c_4^m}{m!}(-1)^m X^m =$$
$$= a_0 + \sum_{m=1}^{\infty} a_m X^m, \quad (47)$$

where

$$a_0 = A + B, \quad a_m = B\frac{c_4^m}{m!}(-1)^m,$$
$$A = c_4 + \beta, \quad B = 1 \mp R_{12},$$
$$Q_2(X) = \mp\beta R_{12}\exp(-c_4X) =$$
$$= \mp\beta R_{12}\sum_{m=0}^{\infty}\frac{c_4^m}{m!}(-1)^m X^m = \mp\sum_{m=0}^{\infty} b_m X^m, \quad (48)$$

where

$$b_m = \beta R_{12}\frac{c_4^m}{m!}(-1)^m,$$
$$R(X) = \beta R_{12}\Theta_c'''\exp(-c_4X) =$$
$$= \beta R_{12}\Theta_c'''\sum_{m=0}^{\infty}\frac{c_4^m}{m!}(-1)^m X^m = \sum_{m=0}^{\infty} d_m X^m, \quad (49)$$

where

$$d_m = \beta R_{12}\Theta_c'''\frac{c_4^m}{m!}(-1)^m.$$

The series (46)–(49) are absolutely convergent for $c_4 > 0$. By substituting (46)–(49) into the original equation (41) and determined the coefficients s_n by the method of undetermined coefficients, we find the values for all the s_n. The coefficients s_0 and s_1 are determined from (46) on the basis of the boundary conditions (42–(45).

Thus, we obtain, for parallel flow

$$s_0 = -1, \quad s_1 = R_{12}, \quad (50)$$

for counterflow

$$s_0 = \Theta_f'', \quad s_1 = R_{12}\Theta_f''. \quad (51)$$

The remaining coefficients are determined from the formula

$$s_{n+1} = \frac{1}{n(n+1)}\left[d_{n-1} - \sum_{i=1}^{n} i s_i a_{n-i} - \sum_{j=i-1}^{n-1} s_j b_{n-(j+1)}\right], \quad n = 1, 2, 3\ldots \quad (52)$$

As before, for $X \geq 0.5$, it is convenient to replace X in the solution (46) by $X - X_0$, it being required that $X - X_0 \leq 0.5$.

From (39) we have

$$\Theta' = \Theta'' \mp R_{21} \exp(c_4 X) \frac{d\Theta''}{dX}. \tag{53}$$

Substituting the value of $d\Theta''/dX$ from (46) into (53), we obtain an expression for the dimensionless temperatures of the gases

$$\Theta' = \Theta'' \mp R_{21} \exp(c_4 X) \sum_{n=1}^{\infty} n s_n X^{n-1}. \tag{54}$$

For the counterflow case, when $t_c''' = t_i'$, which occurs in the majority of practical cases, it follows from (8) that $\Theta_c''' = -1$, and thus, according to (48) and (49), $|b_m| = |d_m|$.

As is clear from what has been said, the equations obtained are used to determine the temperatures of heat transfer agents when the heat transfer coefficients and surface areas are known. The inverse problem is possible, however, if the temperature is known, then, for given heat transfer coefficients, we may determine the heat transfer area, while, for a given heat transfer area, we may determine the local heat transfer coefficient. In the latter cases, however, the matter of finding the desired values reduces to solution of a transcendental equation, and it is therefore convenient to carry out the determination from tables or a nomogram.

NOTATION

c is the an empirical coefficient; F is the heat transfer surface area; k is the overall heat transfer coefficient; $R_{12} = W_1/W_2$, $R_{13} = W_1/W_3$ are the ratios of water equivalents; t is the temperature heat transfer agents; W is the water equivalent; α is the heat transfer coefficient. Subscripts: 1, 2, 3, and also single prime, double prime, and triple prime correspond to the first, second, and third heat transfer agents; i and f are the initial and final values of the quantities; c indicates that the quantity is constant.

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